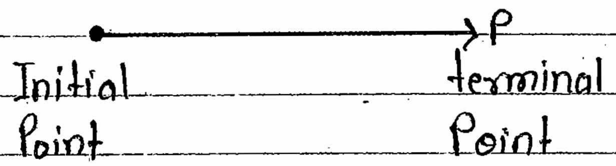
✳ MATHEMATICAL PHYSICS ✳

Vectors

Physical quantities having both magnitude and direction and which follow the vector law of addition are known as vectors.
e.g. velocity, accelaration, force, displacement etc.



$$\vec{OP} = |OP|\,\hat{P}$$

mag. Dir^n

$\hat{OP}$ = Unit Vector along OP

Polar Vectors :-

The vector associated with a linear directional effect are called polar vectors. The examples of polar vectors are force, acceleration, linear velocity, ~~angular~~ linear momentum etc.

Axial Vectors :-

The vectors associated with rotation about an axis are called axial vectors. The examples of axial vectors are: torque, angular velocity, angular momentum etc.

Null Vector :-

Here magnitude is zero, hence initial and terminal point coinside.
Direction = orbitary direction
Displacement vector = Null vector

Reciprocal Vector :-

∵ We know -

$$\vec{a} = |\vec{a}|\,\hat{a}$$

So,

$$\boxed{\vec{a}^{\,-1} = \frac{1}{|\vec{a}|}\,\hat{a}}$$

Negative Vector :-

$$\boxed{-\vec{a} = |\vec{a}|\,(-\hat{a})}$$

Coplanar and Non-Coplanar Vectors :-

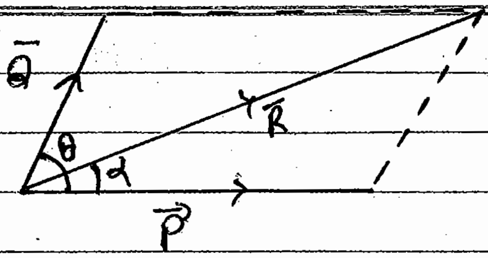
Three or more vectors are said to be coplanar when they are parallel to the same plane. A plane parallel to the system of coplanar vectors is said to be the plane of vectors. Three or more vectors which are not parallel to common plane are said to be non-coplanar.

Unit Vectors :-

"A vector with unit scalar magnitude is defined as unit vector."

Let a vector be $x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Unit Vector} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$$

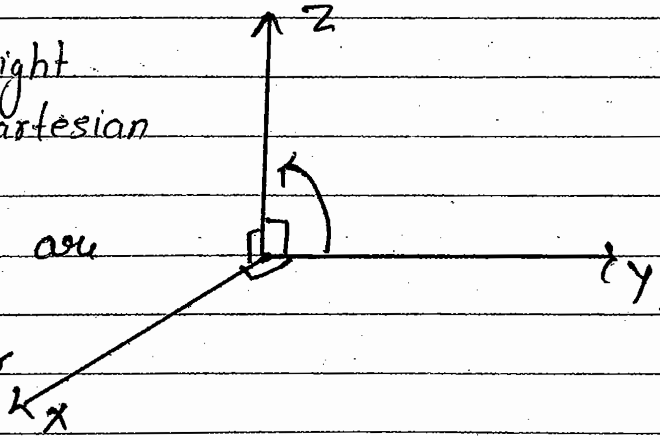


$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\boxed{\sin\alpha = \frac{Q\sin\theta}{R}}$$

* Co-Ordinate System :-

1. Right Handed Cartesian System :-



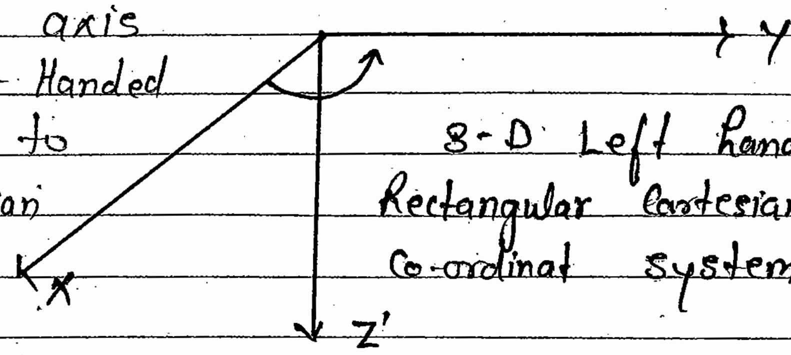
⇒ 3-D ~~Rectangul~~ Right handed Rectangular cartesian coordinate system.

⇒ Here X, Y, Z axis are mutually perpendicular so it is rectangular

⇒ The orientation of X, Y, Z axis follow Right Handed Thumb Rule, so it is Right Handed.

2. Left Handed Co-ordinate system :-

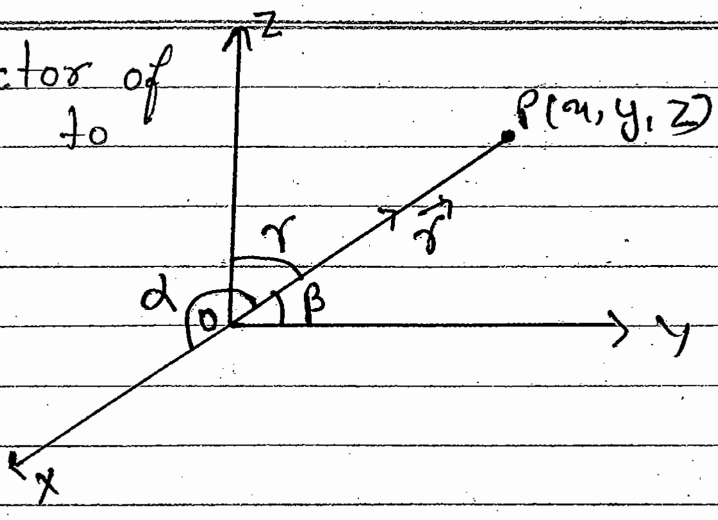
Invert any one axis to convert Right Handed Cartesian system to Left Handed cartesian System.



3-D Left handed Rectangular Cartesian Co-ordinat system.

→ It follow Left Hand Thumb Rule.

$\vec{OP} = \vec{r}$ = Position Vector of point P with respect to origin O.



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{r = \sqrt{x^2 + y^2 + z^2}}$$

Where $\hat{i}, \hat{j}, \hat{k} \rightarrow$ Orthogonal basis

Note :-

$r = \sqrt{x^2+y^2+z^2}$, This formula is applicable only when $\hat{i}, \hat{j}, \hat{k}$ are mutually perpendicular.

Unit Vector Along $\vec{r}$:-

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$

So

$$\boxed{\hat{r} = \frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}}$$

* Direction Cosines of $\vec{r}$:-

If α, β, γ is the angle, which $\vec{r}$ making with X, Y, Z axis, then -

$$l = \cos\alpha = \frac{x}{r}$$

$$m = \cos\beta = y/r$$

$$n = \cos\gamma = z/r$$

$$\boxed{l^2 + m^2 + n^2 = 1}$$

So $\boxed{\hat{r} = l\hat{i} + m\hat{j} + n\hat{k}}$

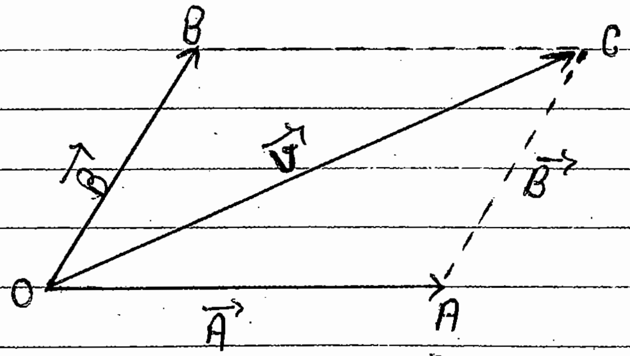
$$\boxed{\hat{r} = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}}$$

Where l, m, n are the direction cosines.

* Addition of Vectors :-

The composition of two displacements of a point has all the characteristics of a summation.

Let us consider two vectors $\vec{A}$ and $\vec{B}$ acting on a point O as shown in figure (a). The resultant effect of the vectors $\vec{A}$ and $\vec{B}$ is same as that the their vector sum $\vec{V}$. The vector $\vec{V}$ is obtained by setting off the vector $\vec{B}$ at the end of $\vec{A}$ and drawing the vector $\vec{V}$ joining the begining of $\vec{A}$ to the end of $\vec{B}$.



So $\vec{V} = \vec{OC} = \vec{OA} + \vec{AC}$

$$\boxed{\vec{V} = \vec{A} + \vec{B}}$$

A similar result is obtained by starting with $\vec{B}$ and setting of the vector $\vec{A}$ on $\vec{B}$.

So,

$$\vec{V} = \vec{OC} = \vec{OB} + \vec{BC} = \vec{B} + \vec{A}$$

$$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$$

Which shows that the sum of two vectors is the diagonal of the parallelogram with the vectors as its sides and this sum obeys the law of commutation according to which the sum of two quantities is independent of the order of the quantities.